Perturbative Expansion in the Galilean Invariant Spin One-Half Chern-Simons Field Theory

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Abstract

A Galilean Chern-Simons field theory is formulated for the case of two interacting spin-1/2 fields of distinct masses M and M'. A method for the construction of states containing N particles of mass M and N' particles of mass M' is given which is subsequently used to display equivalence to the spin-1/2 Aharonov-Bohm effect in the $N = N' = 1$ sector of the model. The latter is then studied in perturbation theory to determine whether there are divergences in the fourth order (one loop) diagram. It is found that the contribution of that order is finite (and vanishing) for the case of parallel spin projections while the antiparallel case displays divergences which are known to characterize the spin zero case in field theory as well as in quantum mechanics.

I. Introduction

The Aharonov-Bohm (AB) effect [1] has been studied extensively in recent years both in the context of quantum mechanics as well as in quantum field theory. As an application of wave mechanics it is customarily idealized to the discussion of the scattering of charged particles from a magnetized filament of arbitrarily small radius. Since the exterior of such a filament is a field free region, there can be no classical force on the particles. The fact that a nontrivial scattering cross section is found thus provides a forceful demonstration of the significance of the vector potential in the quantum mechanical description of scattering.

Although not used in the original AB work the partial wave description of this phenomenon is of considerable interest. For the partial wave $f_m(r)$ the relevant equation is

$$\left[\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + k^2 - \frac{(m+\alpha)^2}{r^2}\right] f_m(r) = 0$$

where α is the flux parameter and $k^2 = 2ME$ with M the particle mass and E its nonrelativistic energy. Standard techniques allow one to obtain that the phase shifts δ_m are given by

$$\delta_m = -\frac{\pi}{2} \mid m+\alpha \mid + \frac{\pi}{2} \mid m \mid$$

so that for small α one finds the nonanalytic form

$$\delta_0 = - \mid \alpha \mid \frac{\pi}{2}.$$

Since in the $m = 0$ partial wave the potential is proportional to α^2, this suggests that a perturbative approach may encounter considerable difficulty [2]. Aharonov *et al.* demonstrated the existence of singularities in such an expansion using as a model an impenetrable solenoid of finite radius R[3]. Although the limit $R \to 0$ was found to yield the usual AB scattering amplitude, to any finite order in α the solution consisted of a complicated expansion in powers of $\alpha \ell n(kR/2)$.

Similar studies have been carried out in the context of field theory. One prerequisite to such a study was the construction of the so-called pure Chern-Simons (or photonless) gauge theory as carried out by this writer[4]. It was subsequently shown [5] that the Galilean limit of such a theory allows one to formulate what is the only Galilean invariant gauge theory known at this time. It is in fact the field theory of the AB effect, a result which has made possible the study of this phenomenon in perturbation theory. Calculations which have been carried out for spinless particles have found that unless an additional (contact) interaction is introduced into the theory, divergences similar to those encountered in quantum mechanical calculations will occur [6].

An extension of the AB effect to include spin has also been carried out in the context of the Dirac equation[7]. This has led to a recognition of the fact that there must exist solutions of the wave equation which are singular at the origin in the case that the spin orientation of the scattered particles is such that the Zeeman interaction is attractive. However, a remarkable feature of the spin-1/2 case is the absence of divergences of the type which characterize perturbation theory in the spinless case[8]. Clearly it would be of interest to determine whether the corresponding spin-1/2 Galilean field theory is also free of perturbative singularities. It is in fact the goal of the present work to demonstrate this result to fourth order in the coupling (second order in α).

In the following section a brief summary of the necessary field theoretic tools is given, including a demonstration of how one proceeds from the Hamiltonian of the field theory to a wave equation in the two particle sector. Section III introduces the fourth order diagram and carries out its evaluation in the limit of very large mass for one of the two particles participating in the scattering process. Using this result it is possible to carry out in section IV the evaluation of the scattering amplitude in the general mass case and to determine the effect of the particle spins on the overall result. Some concluding remarks are offered in section V.

II. Spin-1/2 Chern-Simons Galilean Field Theory

Since the case of a spinless field in interaction with a Chern-Simons field described by three components ϕ and ϕ_i (i=1,2) has been discussed in some detail in ref. 5, it will be sufficient to present a somewhat brief review of this subject, giving principal

attention to those features associated with the spin of the charged particle. It is convenient to begin with a single spin-1/2 particle of mass M. As shown by Lévy-Leblond [9] a first order wave equation requires a four component spin-1/2 field operator in three spatial dimensions. For two spatial dimensions, however, a two component field operator suffices for the description of a single spin component (just as in the case of the Dirac equation in two spatial dimensions). For the free field case such an operator can be taken to satisfy the equation

$$\left[(1/2)(1+\sigma_3)i\frac{\partial}{\partial t}+i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}+M(1-\sigma_3)\right]\psi=0 \tag{1}$$

where σ_3 is the usual third Pauli matrix while the matrices $\sigma_i (i = 1, 2)$ are the set $(\sigma_1, s\sigma_2)$ where s is twice the spin projection (+1 for spin "up" and -1 for spin "down").

The combined system of interacting spinor and Chern-Simons field can be described by the Lagrangian

$$\begin{aligned}\mathcal{L} \;&=\; \psi^\dagger\left[\frac{1}{2}(1+\sigma_3)(i\frac{\partial}{\partial t}-g\phi)+i\boldsymbol{\sigma}\cdot(\boldsymbol{\nabla}-ig\boldsymbol{\phi})+M(1-\sigma_3)\right]\psi\\ &\quad -\frac{1}{2}\phi\boldsymbol{\nabla}\times\boldsymbol{\phi}-\frac{1}{2}\boldsymbol{\phi}\times\boldsymbol{\nabla}\phi-\frac{1}{2}\boldsymbol{\phi}\times\frac{\partial}{\partial t}\boldsymbol{\phi}\end{aligned}$$

which implies the equations of motion

$$-\boldsymbol{\nabla}\times\boldsymbol{\phi}=g\rho \tag{2}$$

$$\epsilon_{ij}\left[\frac{\partial}{\partial t}\phi_j+\nabla_j\phi\right]=gj_i \tag{3}$$

$$\left[\frac{1}{2}(1+\sigma_3)(i\frac{\partial}{\partial t}-g\phi)+i\boldsymbol{\sigma}\cdot(\boldsymbol{\nabla}-ig\boldsymbol{\phi})+M(1-\sigma_3)\right]\psi=0. \tag{4}$$

where

$$\rho=\psi^\dagger\frac{1}{2}(1+\sigma_3)\psi$$

$$j_i=\psi^\dagger\sigma_i\psi.$$

In the radiation gauge

$$\boldsymbol{\nabla}\cdot\boldsymbol{\phi}=0,$$

Eq.(2) has the solution

$$\phi_i = -g\epsilon_{ij}\nabla_j \int d^2x'\ \mathcal{D}(x-x')\rho(x') \tag{5}$$

where $\mathcal{D}(x)$ is defined by

$$-\boldsymbol{\nabla}^{\mathbf{2}}\mathcal{D}(x) = \delta(\mathbf{x})$$

or more explicitly by

$$\mathcal{D}(x) = -\frac{1}{4\pi}\ell n\mathbf{x}^{\mathbf{2}} + \text{constant}.$$

Upon insertion of (5) into (3) one obtains for $\phi(x)$ the result

$$\phi(x) = g\int d^2x\ \mathbf{j}(x') \times \boldsymbol{\nabla}\mathcal{D}(x-x'). \tag{6}$$

It is worth emphasizing here the well known fact that there are no independent degrees of freedom associated with the gauge fields (ϕ, ϕ_i) since they are expressible as explicit functions of the charged spin-1/2 fields[4] as is clearly seen from Eqs.(5) and (6).

Also of interest is a more explicit display of the content of Eq.(4). Denoting the upper and lower components by φ and χ respectively, one has for φ the result

$$(i\frac{\partial}{\partial t} - g\phi)\varphi = (\Pi_1 - is\Pi_2)\chi \tag{7}$$

where $\Pi_i = -i\nabla_i - g\phi_i$. Because of the presence of the time derivative in (7) it is a true equation of motion as opposed to the equation for χ which is of the form

$$2M\chi = (\Pi_1 + is\Pi_2)\varphi.$$

Thus χ is a dependent field operator which is locally defined in terms of φ and the gauge field operators.

Application of the action principle allows one to infer the equal time anticommutation relation

$$\{\varphi(x), \varphi^\dagger(x')\} = \delta(\mathbf{x}-\mathbf{x}') \tag{8}$$

and the form of the conserved mass operator of the theory

$$\mathcal{M} = M\int d^2x\ \varphi^\dagger\varphi.$$

Since it will be convenient to consider the AB scattering of dissimilar spin-1/2 particles in this work, the foregoing analysis will henceforth be understood to include two fields ψ and ψ' of masses M and M' respectively, each of which has identical coupling to the Chern-Simons gauge field, and (possibly different) spin projections s and s'. Thus the commutation relation (8) is assumed to apply to the fields φ and φ' separately while the mass operator becomes

$$\mathcal{M} = \int d^2x \, [M\varphi^\dagger\varphi + M'\varphi'^\dagger\varphi'].$$

The form of the interaction implies the existence of an additional (global) symmetry which leads to the conclusion that each of the two terms in $\mathcal{M}$ is separately conserved. This allows the states of the system to be divided into sectors each of which is characterized by non-negative integers N and N' which denote the numbers of particles of masses M and M' respectively. These states can be denoted by $|N, N' >$ and are constructed according to

$$\begin{aligned} |N, N' > \;\; &= \;\; \int d^2x_1 \ldots d^2x_N d^2x'_1 \ldots d^2x'_{N'} \, \varphi^\dagger(x_1) \ldots \varphi^\dagger(x_N)\varphi'^\dagger(x'_1) \\ & \quad \ldots \varphi'^\dagger(x'_{N'}) f(x_1, \ldots x_N; x'_1, \ldots x'_{N'})|0 > \end{aligned}$$

where $|0 >$ is the vacuum or zero particle state

$$\mathcal{M}|0 >= 0 \tag{9}$$

and $f(x_1, \ldots x_N; x'_1, \ldots x'_{N'})$ is the $N + N'$ particle wave function. The consistency of Eq.(9) clearly requires that φ and φ' annihilate the vacuum, i.e.,

$$\varphi(x)|0 >= 0$$

$$\varphi'(x)|0 >= 0.$$

The energy operator is inferred to have the form

$$\mathcal{H} = \int d^2x \, \left[\frac{1}{2M}\varphi^\dagger(\Pi_1 + is\Pi_2)(\Pi_1 - is\Pi_2)\varphi + \frac{1}{2M'}\varphi'^\dagger(\Pi_1 + is'\Pi_2)(\Pi_1 - is'\Pi_2)\varphi'\right]$$

and allows the formulation of the eigenvalue equation

$$\mathcal{H}|N, N' >= E|N, N' > . \tag{10}$$

One solves Eq.(10) by considering separately the various combinations of N and N'. Thus one clearly has the vacuum state for $N = N' = 0$ while the choices $N = 1, N' = 0$ and $N = 0, N' = 1$ yield the trivial results

$$(E + \frac{1}{2M}\nabla^2)f(\mathbf{x}) = 0$$

and

$$(E + \frac{1}{2M'}\nabla'^2)f(\mathbf{x}') = 0$$

respectively. The case $N = N' = 1$ is the sector which describes the AB scattering of dissimilar fermions. Application of Eq.(10) in this case is found to imply the wave equation

$$
\begin{aligned}
&Ef(\mathbf{x},\mathbf{x}') = \\
&\quad -\left\{\frac{1}{2M}\left[\left[\nabla_i + i\frac{g^2}{2\pi}\epsilon_{ij}\frac{(x-x')_j}{(x-x')^2}\right]^2 - sg^2\delta(\mathbf{x}-\mathbf{x}')\right]\right. \\
&\quad \left. +\frac{1}{2M'}\left[\left[\nabla_i - i\frac{g^2}{2\pi}\epsilon_{ij}\frac{(x-x')_j}{(x-x')^2}\right]^2 - s'g^2\delta(\mathbf{x}-\mathbf{x}')\right]\right\} f(\mathbf{x},\mathbf{x}') \qquad (11)
\end{aligned}
$$

with similar results following for the cases $N = 2, N' = 0$ and $N = 0, N' = 2$ which describe the AB scattering of identical particles of masses M and M' respectively. Worth noting in (11) is the explicit appearance of the spin dependent terms proportional to s and s'. These are of the contact type and describe the Zeeman interaction of the magnetic moments of the particles. Since the system described by Eq.(11) allows one to avoid the inessential complication of the Pauli principle and has the further advantage of allowing the simultaneous consideration of parallel and antiparallel spins, the remainder of this paper will focus exclusively on the unequal mass case.

One solves Eq.(11) in the usual way by separation into the center-of-mass coordinates and the relative coordinates $\mathbf{r} = \mathbf{x} - \mathbf{x}'$. This leads to the reduced equation for the wave function $f(\mathbf{r})$

$$
\left[\left[\nabla_i - i\alpha\epsilon_{ij}r_j/r^2\right]^2 - \alpha\mu\left(\frac{s}{M} + \frac{s'}{M'}\right)\frac{1}{r}\delta(r) + k^2\right] f(\mathbf{r}) = 0 \qquad (12)
$$

where k^2 is the wave number in the center-of-mass frame, μ is the reduced mass

$$
\mu = \frac{MM'}{M+M'},
$$

and use has been made of the definition $g^2/2\pi = \alpha$. (It is to be noted that negative values of α can be obtained by consideration of a Chern-Simons theory which is identical in all respects except for a change in the sign of the terms in the Lagrangian which are quadratic in the gauge fields.) The result (12) in the case $s = s'$ reduces to

$$
\left[\left[\nabla_i - i\alpha\epsilon_{ij}r_j/r^2\right]^2 - \alpha s\frac{1}{r}\delta(r) + k^2\right] f(\mathbf{r}) = 0
$$

and is the basis for the quantum mechanical description of spin-1/2 AB scattering [7]. Since it is in the unique case $s = s'$ that one has a divergence free perturbation expansion in quantum mechanics [8], the goal of the remainder of this paper is to provide a corresponding demonstration that to the one loop order it is only in this

special case that the corresponding field theoretic perturbation calculation is also divergence free.

III. Fourth Order AB Scattering

In order to carry out the desired field theoretic perturbation expansion it is necessary to prescribe the propagators and vertices associated with the model. The free field equation (1) for the field ψ implies the momentum space equation for the mass M fermion propagator

$$\left[\frac{1}{2}(1+\sigma_3)E - \boldsymbol{\sigma}\cdot\mathbf{p} + M(1-\sigma_3)\right]G(\mathbf{p},E) = 1$$

which has the solution

$$G(\mathbf{p},E) = \left[M(1+\sigma_3) + \boldsymbol{\sigma}\cdot\mathbf{p} + \frac{1}{2}(1-\sigma_3)E\right][2ME - p^2 + i\epsilon]^{-1}.$$

With respect to the gauge fields it is convenient to introduce a notation such that ϕ^α denotes the set (ϕ^0, ϕ^i) with ϕ^0 identified with ϕ. By standard means one then infers [4] that the Chern-Simons propagator is given by

$$\mathcal{G}^{\alpha\beta}(\mathbf{k}) = i\epsilon^{\alpha\beta j}k_j\frac{1}{k^2}.$$

Inspection of the form of $\mathcal{L}$ allows one to infer the fact that the vertex matrices Γ^α are given by the set $(\frac{1}{2}(1+\sigma_3), \sigma_i)$.

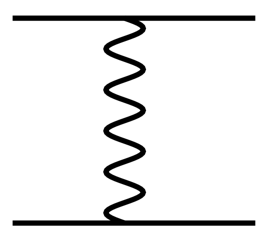

Figure 1: The second order scattering diagram

To second order in g one has only the single ϕ exchange diagram displayed in Fig.1. It is proportional to

$$\left[\frac{1}{2}(1+\sigma_3)^{(1)}\sigma_i^{(2)} - \frac{1}{2}(1+\sigma_3)^{(2)}\sigma_i^{(1)}\right]\epsilon_{ij}k_j\frac{1}{k^2} \tag{13}$$

where k_i is the momentum transfer for the fermion of mass M of incoming momentum p_i and outgoing momentum p'_i. A superscript notation has been used to specify the matrices of the two particles so that (1) and (2) refer respectively to mass M and M' particles. It is to be noted that (13) is to be evaluated between $u(\mathbf{p})^{(1)}u(-\mathbf{p})^{(2)}$ and $u^*(\mathbf{p}')^{(1)}u^*(-\mathbf{p}')^{(2)}$ (working in the center-of-mass frame) where the u's are the relevant free particle spinors. From (1) one infers these to be of the form

$$u(\mathbf{p}) = \begin{pmatrix} 1 \\ \frac{p_1 + isp_2}{2M} \end{pmatrix}.$$

Denoting the angle between $\mathbf{p}$ and $\mathbf{p}'$ by θ one finds that an evaluation of (13) between the indicated spinors yields a scattering amplitude proportional to

$$\frac{g^2}{\mu\ \sin(\theta/2)}[\cos(\theta/2) - i\mu(\frac{s}{M} + \frac{s'}{M'})\sin(\theta/2)]$$

which reduces in the $s = s'$ case to

$$\frac{g^2}{\mu\ \sin(\theta/2)} e^{-is\theta/2}$$

in agreement with the order α result which one obtains from an expansion of the exact scattering amplitude [7]. Similar results have been obtained to this order using covariant perturbation theory in the infinite M' limit [10].

Since the spin-1/2 scattering amplitude is known [7] to have no $O(\alpha^2)$ corrections, one seeks to verify that the fourth order in g result is both finite and null. The specific diagram is displayed in Fig. 2 and is formally given by

$$\begin{aligned} &\int \frac{dk\ dE}{(2\pi)^3} \left\{ \Gamma^\alpha [M(1+\sigma_3) - \boldsymbol{\sigma}\cdot\mathbf{k} - \frac{1}{2}E(1-\sigma_3)]\Gamma^\beta \right\}^{(1)} \\ &\left\{ \Gamma^\kappa [M'(1+\sigma_3) + \boldsymbol{\sigma}\cdot\mathbf{k} + \frac{1}{2}(E + \frac{p^2}{2\mu})(1-\sigma_3)]\Gamma^\lambda \right\}^{(2)} \\ &\mathcal{G}_{\beta\lambda}(\mathbf{p}+\mathbf{k})\mathcal{G}_{\alpha\kappa}(\mathbf{p}'+\mathbf{k}) \frac{1}{-2ME - k^2 + i\epsilon}\ \frac{1}{2M'(E + \frac{p^2}{2\mu}) - k^2 + i\epsilon}. \end{aligned} \tag{14}$$

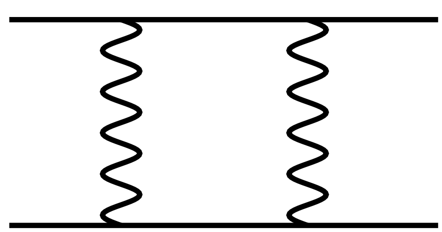

Figure 2: The fourth order scattering diagram

Although the integration over E is superficially linearly divergent, in actuality the divergence is only a logarithmic one. This can be seen by noting that the antisymmetry of $\mathcal{G}_{\alpha\beta}$ in α and β implies that at least one of the factors of E in the numerator of (14) is necessarily multiplied by the product of $(1+\sigma_3)$ with $(1-\sigma_3)$ and thus

vanishes. The remaining divergence must be regulated in a Galilean invariant manner and requires some care.

It will be convenient to classify contributions to (14) according to whether the vertex indices are spatial ($\alpha = i$) or temporal ($\alpha = 0$). Referring to (14) one sees that the case α being a temporal (spatial) index requires that κ be a spatial (temporal) one (and similarly for β and λ). Thus it follows that (14) decomposes in a natural way according to whether a given fermion line has only temporal vertices, only spatial vertices, or mixed vertices. Contribution 1 will thus be that part of (14) which has purely spatial vertices in the mass M propagator (temporal vertices in the mass M' propagator), contribution 2 will be the corresponding case in which M and M' are exchanged, and contribution 3 will be the two parts in which mixed vertices occur. It is to be noted that there is no divergence in the E integration for contribution 3.

In the limit of large M' only contribution 1 survives. This is a consequence of the fact that terms of the form $(1+\sigma_3)\sigma_i u(\mathbf{p})^{(2)}$ vanish for large M'. It is worth noting that the surviving term is precisely what one would consider in the case of scattering from a fixed flux tube source. Regularization is accomplished by the replacement

$$\frac{1}{-2ME - k^2 + i\epsilon} \rightarrow \frac{1}{-2ME - k^2 + i\epsilon} - \frac{1}{-2M(E+U) - k^2 + i\epsilon}.$$

and subsequently taking the limit $U \rightarrow \infty$. This is a Galilean invariant regularization scheme since it consists of the addition of an internal energy term to the Galilean invariant quantity in the fermion propagator. Upon performing the integration over E one finds that (14) reduces to

$$\begin{aligned} -i\int \frac{dk}{(2\pi)^2}\, \frac{1}{(\mathbf{p}+\mathbf{k})^2}\frac{1}{(\mathbf{p}'+\mathbf{k})^2}\frac{1}{p^2-k^2+i\epsilon}\boldsymbol{\sigma}\times(\mathbf{p}'+\mathbf{k}) \\ \left[M(1+\sigma_3) - \boldsymbol{\sigma}\cdot\mathbf{k} + \frac{1}{2}(1-\sigma_3)(p^2/2M)\right]\boldsymbol{\sigma}\times(\mathbf{p}+\mathbf{k}), \end{aligned} \tag{15}$$

which is seen by simple power counting to be finite. Thus the AB scattering of spin-1/2 particles by a fixed flux tube is finite in this order. This is to be contrasted with the spin zero case which is, of course, rendered finite at the one loop level only by the addition of a suitable contact term.

It remains to be seen whether the result (15) vanishes on the "internal energy shell", i.e., when

$$\left[\frac{1}{2}(1+\sigma_3)(p^2/2M) - \boldsymbol{\sigma}\cdot\mathbf{p} + M(1-\sigma_3)\right]u(\mathbf{p}) = 0. \tag{16}$$

On applying (16) it is found that (15) becomes

$$-i\sigma_i \int \frac{dk}{(2\pi)^2} \frac{1}{p^2 - k^2 + i\epsilon} \left\{ \frac{1}{2} \left[\frac{(p+k)_i}{(\mathbf{p}+\mathbf{k})^2} + \frac{(p'+k)_i}{(\mathbf{p}'+\mathbf{k})^2} \right] \right.$$
$$\left. + \epsilon_{ij} \left[(p'+k)_j (\mathbf{p}\times\mathbf{k}) + (p+k)_j (\mathbf{p}'\times\mathbf{k}) \right] \frac{1}{(\mathbf{p}+\mathbf{k})^2} \frac{1}{(\mathbf{p}'+\mathbf{k})^2} \right\}.$$

Symmetry considerations imply that the integral in this expression can be written in terms of two scalar functions $A(\mathbf{p},\mathbf{p}')$ and $B(\mathbf{p},\mathbf{p}')$ as

$$A(\mathbf{p},\mathbf{p}')(p+p')_i + B(\mathbf{p},\mathbf{p}')\epsilon_{ij}(p-p')_j(\mathbf{p}\times\mathbf{p}'). \tag{17}$$

Since the matrix element of σ_i is given by

$$u^*(\mathbf{p}')\sigma_i u(\mathbf{p}) = \frac{1}{2M} \left[(p+p')_i + is\epsilon_{ij}(p-p')_j \right],$$

it is necessary to contract (17) with $(p+p')_i$ and $\epsilon_{ij}(p-p')_j$. In the former case one obtains after some algebra that

$$\begin{aligned} A(\mathbf{p},\mathbf{p}')(\mathbf{p}+\mathbf{p}')^2 - 2B(\mathbf{p},\mathbf{p}')(\mathbf{p}\times\mathbf{p}')^2 \;\; &= \;\; \int \frac{dk}{(2\pi)^2} \left\{ \frac{4(\mathbf{p}\times\mathbf{k})(\mathbf{p}'\times\mathbf{k})}{(\mathbf{p}+\mathbf{k})^2(\mathbf{p}'+\mathbf{k})^2} \frac{1}{p^2-k^2+i\epsilon} \right. \\ &\left. + \frac{(\mathbf{p}+\mathbf{k})\cdot(\mathbf{p}'+\mathbf{k})}{(\mathbf{p}+\mathbf{k})^2(\mathbf{p}'+\mathbf{k})^2} + \frac{1}{p^2-k^2+i\epsilon} \right\} \end{aligned} \tag{18}$$

Upon comparison with ref. 6 one sees that the result (18) plus the corresponding expression with $\mathbf{p}' \to -\mathbf{p}'$ is proportional to the scattering amplitude for identical spinless particles for the case in which a contact term of the appropriate magnitude has been included. Since the latter was specifically constructed so as to give a vanishing result, it is plausible that the right hand side of (18) also vanishes. This can in fact be verified by direct calculation. Worth emphasizing here is the fact that the calculation of ref. 6 required that the noncovariant cutoffs of the k integrals for both the Chern-Simons interaction and the contact term be taken to be the same. No such assumption is required here since the integral in (15) is finite.

To complete the argument it is also necessary to verify that contraction of $\epsilon_{ij}(p-p')_j$ with (17) gives a vanishing result. Calculation shows that one again obtains the right hand side of (18) up to an overall kinematic factor, thereby establishing that the one loop correction vanishes in the spin-1/2 case for $M' \to \infty$. The removal of this latter condition is accomplished in the following section.

IV The General Mass Case

In order to consider the case of AB scattering with general masses M and M' one returns to (14). Upon regulating the divergence in the energy integration in

contributions 1 and 2 as previously described one obtains

$$\frac{-i}{M+M'} \int \frac{dk}{(2\pi)^2} \frac{1}{(\mathbf{p}+\mathbf{k})^2} \frac{1}{(\mathbf{p}'+\mathbf{k})^2} \frac{1}{p^2-k^2+i\epsilon}$$
$$\Big\{M'\Big[\boldsymbol{\sigma}\times(\mathbf{p}'+\mathbf{k})\big[M(1+\sigma_3)-\boldsymbol{\sigma}\cdot\mathbf{k}+\frac{1}{2}(1-\sigma_3)\left(\frac{p^2}{2\mu}-\frac{k^2}{2M'}\right)\big]\boldsymbol{\sigma}\times(\mathbf{p}+\mathbf{k})\Big]^{(1)}$$
$$\frac{1}{2}(1+\sigma_3)^{(2)}+M\frac{1}{2}(1+\sigma_3)^{(1)}\Big[\boldsymbol{\sigma}\times(\mathbf{p}'+\mathbf{k})\big[M'(1+\sigma_3)+\boldsymbol{\sigma}\cdot\mathbf{k}+\left(\frac{p^2}{2\mu}-\frac{k^2}{2M}\right)\big]$$
$$\boldsymbol{\sigma}\times(\mathbf{p}+\mathbf{k})\Big]^{(2)}-\frac{1}{2}\Big[\frac{1}{2}(1+\sigma_3)\big[M(1+\sigma_3)-\boldsymbol{\sigma}\cdot\mathbf{k}\big]\boldsymbol{\sigma}\times(\mathbf{p}+\mathbf{k})\Big]^{(1)}$$
$$\Big[\boldsymbol{\sigma}\times(\mathbf{p}'+\mathbf{k})\big[M'(1+\sigma_3)+\boldsymbol{\sigma}\cdot\mathbf{k}\big]\frac{1}{2}(1+\sigma_3)\Big]^{(2)}$$
$$-\frac{1}{2}\Big[\boldsymbol{\sigma}\times(\mathbf{p}'+\mathbf{k})\big[M(1+\sigma_3)-\boldsymbol{\sigma}\cdot\mathbf{k}\big]\frac{1}{2}(1+\sigma_3)\Big]^{(1)}$$
$$\Big[\frac{1}{2}(1+\sigma_3)\big[M'(1+\sigma_3)+\boldsymbol{\sigma}\cdot\mathbf{k}\big]\boldsymbol{\sigma}\times(\mathbf{p}+\mathbf{k})\Big]^{(2)}\Big\}. \tag{19}$$

The integrand of this expression contains four separate terms, of which the first is the type 1 contribution, the second is the type 2, and the last two are the (mixed) type 3 contribution. Considerable simplification of the former is achieved by applying the condition (16) and the corresponding one for the mass M' part together with the result established in the preceding section concerning the vanishing of the type 1 (type 2) contribution in the infinite M' (infinite M) limit. One finds that the type 1 and type 2 contributions to the curly bracket in Eq.(19) thereby reduce to

$$(p^2-k^2)\frac{1}{2}(1+\sigma_3)^{(1)}\frac{1}{2}(1+\sigma_3)^{(2)}$$
$$\frac{1}{2}\left\{[\boldsymbol{\sigma}\times(\mathbf{p}'+\mathbf{k})\boldsymbol{\sigma}\times(\mathbf{p}+\mathbf{k})]^{(1)}+[\boldsymbol{\sigma}\times(\mathbf{p}'+\mathbf{k})\boldsymbol{\sigma}\times(\mathbf{p}+\mathbf{k})]^{(2)}\right\}$$

which can be written as

$$(p^2-k^2)\frac{1}{2}(1+\sigma_3)^{(1)}\frac{1}{2}(1+\sigma_3)^{(2)}\left[(\mathbf{p}+\mathbf{k})\cdot(\mathbf{p}'+\mathbf{k})-\frac{i}{2}(s+s')(\mathbf{p}+\mathbf{k})\times(\mathbf{p}'+\mathbf{k})\right].$$

The type 3 contributions also undergo considerable simplification when (16) is invoked. One finds that these reduce to

$$-\frac{1}{2}\,\frac{1}{2}(1+\sigma_3)^{(1)}\frac{1}{2}(1+\sigma_3)^{(2)}\Big\{[2\mathbf{k}\times\mathbf{p}-is(\mathbf{p}+\mathbf{k})^2]^2[2\mathbf{p}'\times\mathbf{k}-is'(\mathbf{p}'+\mathbf{k})^2]$$
$$+[2\mathbf{k}\times\mathbf{p}-is'(\mathbf{p}+\mathbf{k})^2][2\mathbf{p}'\times\mathbf{k}-is(\mathbf{p}'+\mathbf{k})^2]\Big\},$$

which can be written as

$$\frac{1}{2}(1+\sigma_3)^{(1)}\frac{1}{2}(1+\sigma_3)^{(2)}\Big\{4(\mathbf{p}\times\mathbf{k})(\mathbf{p}'\times\mathbf{k})+ss'(\mathbf{p}+\mathbf{k})^2(\mathbf{p}'+\mathbf{k})^2$$
$$-i(s+s')[(\mathbf{k}\times\mathbf{p}')(\mathbf{p}+\mathbf{k})^2-(\mathbf{k}\times\mathbf{p})(\mathbf{p}'+\mathbf{k})^2]\Big\}.$$

It is seen by inspection that the terms linear in s and s' vanish upon doing the angular integration so that one obtains upon combining all these results the reduction of (19) to

$$-\frac{i}{M+M'}\frac{1}{2}(1+\sigma_3)^{(1)}\frac{1}{2}(1+\sigma_3)^{(2)}\int\frac{dk}{(2\pi)^2}\left\{\frac{4(\mathbf{p}\times\mathbf{k})(\mathbf{p}'\times\mathbf{k})}{(\mathbf{p}+\mathbf{k})^2(\mathbf{p}'+\mathbf{k})^2}\frac{1}{p^2-k^2+i\epsilon}\right.$$
$$\left.+\frac{ss'}{p^2-k^2+i\epsilon}+\frac{(\mathbf{p}+\mathbf{k})\cdot(\mathbf{p}'+\mathbf{k})}{(\mathbf{p}+\mathbf{k})^2(\mathbf{p}'+\mathbf{k})^2}-\frac{i}{2}(s+s')\frac{(\mathbf{p}+\mathbf{k})\times(\mathbf{p}'+\mathbf{k})}{(\mathbf{p}+\mathbf{k})^2(\mathbf{p}'+\mathbf{k})^2}\right\}.$$

Again it can be shown that the term which is linear in $s+s'$ vanishes upon performing the angular integration. This leaves one with a result which is remarkably similar to (18). Since the latter is known to vanish, one readily finds that the one loop correction to spin-1/2 AB scattering is given by

$$\frac{-i}{M+M'}\frac{1}{2}(1+\sigma_3)^{(1)}\frac{1}{2}(1+\sigma_3)^{(2)}\int\frac{dk}{(2\pi)^2}\frac{(ss'-1)}{(p^2-k^2+i\epsilon)}$$

which is the final result of this calculation. For $ss'=1$ (i.e., parallel spins) the result is thus seen to be both finite and vanishing to order α^2. In this case one has equivalence to the $s=s'$ limit of Eq.(12) which is known [8] to have a divergence free perturbative expansion. Conversely, for the antiparallel spin configuration one has either a magnetic moment which vanishes ($M=M'$) or one which has a g-factor less than 2 (general M, M') and is thus not equivalent to the spin-1/2 AB scattering system studied in ref. 7. In each of these latter cases one expects divergences in perturbative calculations in agreement with the results obtained here.

V Conclusion

This work has succeeded in restoring a certain symmetry between spin zero and spin-1/2 work on the AB effect. While the quantum mechanical AB effect had been solved for both the scalar [1] and spinor [7] cases and their perturbative expansions studied in both applications [2,3,8], only the scalar theory had been studied previously as a perturbation expansion in field theory. Although the technical complications associated with the matrix algebra are quite significant in the spin-1/2 field theory, it has in fact been found possible to carry through the calculation of the AB scattering amplitude to fourth order in the coupling constant and thereby reestablish the aforementioned balance between the scalar and spinor theories.

It is certainly gratifying that the results of this study conform with those which have been found in ref.8. Beyond that, however, is the very useful set of rewards which have followed from the use of nonidentical particles in this study. First of all it allowed one to begin the calculation with the much more manageable examination of AB scattering from a fixed flux tube (i.e., the $M' \to \infty$ limit), obtaining in the process a result which greatly facilitated the treatment of the more general case. Second, it allowed one to avoid the extraneous complication arising from the Pauli principle. Finally, and perhaps most significant of all, it allowed the simultaneous consideration of the parallel and antiparallel spin cases. It has been found that in the former case the result is both finite and vanishing at the fourth order while in the latter one encounters the divergences known to characterize the spin zero theory. All of these results are in conformity with calculations which have been carried out in the context of quantum mechanics.

Finally, mention should be made of the fact that it must be regarded as encouraging that calculations such as those presented here can be effectively carried out. This is a significant point since the spin-1/2 theory has a crucial advantage over the corresponding scalar theory by virtue of its being finite in perturbation theory without the *ad hoc* inclusion of additional coupling terms. As has already been mentioned, the fact that the spin-1/2 theory already implies such contact terms through the magnetic moment interaction serves to eliminate ambiguities in the regularization of divergent integrals. It may thus be possible that such features will cause spin-1/2 perturbative calculations to see greater application in the future.

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